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INTEREST RATE DISTRIBUTIONS, YIELD CURVE MODELLING AND MONETARY POLICY

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Abstract

The way in which countries conduct monetary policy significantly affects the distribution of their short interest rates and hence the pricing of bonds further along the yield curve. This paper examines the impact of monetary policy on the higher moments and persistence of interest rate changes. We present a simple version of an analytic model of interest rate setting by monetary authorities and show that this can mimic properties of interest rates that standard models tend to fit badly.

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1 Introduction

1.1 Sources of Interest Rate Changes

In many financial applications, one can afford to be agnostic about the ultimate source of asset price changes. Widely applied finance models such as the CAPM and APT derive implications for relative asset prices from assumptions about the joint distributions of asset returns. Arbitrage models such as the Black-Scholes model of European option valuation proceed from an even tighter distributional assumption namely that changes in the prices of the cash security and the option are perfectly locally correlated.

What one might call the 'statistical approach,' of concentrating exclusively on the joint distribution of asset returns while ignoring the underlying causes of asset price changes, has its limits, however. In government bond markets, all prices ultimately depend on the distribution of the short-term interest rate[1] which, in turn, is largely determined by the actions of the monetary authorities. Understanding the links between policy and interest rate distributions is important because, when policies shift, valuation must be performed before a usable amount of data has accumulated under the new regime.

1.2 Rate Pegging and Leptokurtosis

In this paper, we focus on two aspects of monetary policy and two corresponding features of interest rate distributions. The first is the practice of many monetary authorities of pegging a reference rate at the short end of the yield curve and periodically adjusting it in discrete jumps. For example, in Germany, the reference rate is a repo rate, while in the UK it is the Bank of England's band 1 stop rate. Both rates are held constant for periods of time and the market regards changes as indicating significant policy shifts. By contrast, the US reference rate, the rate on Federal funds, fluctuates around intervention trigger levels set by the authorities.

[1] Note that this distribution may depend on multiple state variables, not just on the level of the short rate. Also, note that strictly speaking, it is, of course, the risk adjusted distribution of the short rate that matters.

An important implication of pegging the reference interest rate is that short-term interest rate changes become increasingly leptokurtic, both conditionally and unconditionally, as one considers higher frequency data. Typical term structure models presume that short-term interest rates follow diffusion processes. It is, of course, possible to specify diffusion processes with highly leptokurtic unconditional distributions but this is likely to require fairly extreme parameter values. Conditionally, all diffusion processes are locally Gaussian in their increments, however. So, as the frequency of data increases, returns with jumps exhibit quite different distributional properties from those generated by diffusions processes.

1.3 Persistence

The second feature of monetary policy on which we shall focus is the resolution that central banks exhibit in their reactions to inflationary shocks. We shall argue that such persistence will show up as persistence in shocks to interest rates. Clearly, monetary authorities in different countries differ greatly in their 'reaction functions', and in particular in the speed with which they relax the tightening or loosening of monetary policy that they adopt in the face of a monetary shock. Numerous studies have examined central bank reaction functions. Among many examples, one might mention McCallum (1994) who examines the constraints on interest rate setting imposed by the need to maintain banking sector stability; Dotsey and King (1986) who look at interest rate setting under differential information; and Siegel (1983) who examines operational interest rate rules.

Few studies, however, have stressed the important implications of central bank reaction functions for the distribution of short interest rates. Stochastic models of interest rates generally have parameters that describe (i) the long run mean interest rate level, and (ii) how rapidly interest rates return to this mean after a shock. (the third crucial element in such models is, of course, the parameter or parameters that describe instantaneous volatility.) Both mean and 'reversion rate' parameters are crucial for many fixed income derivative pricing problems. The rate of reversion to normal interest rate levels directly reflects the speed with which monetary authorities relax their reaction to monetary shocks. Exercise of resolute monetary policy is likely to be associated with slower reversion to 'normal' interest rate levels and hence greater persistence.

1.4 The Contribution of the Paper

In this paper, we begin by discussing the techniques of monetary control employed in Germany, the UK and the US. We contrast the appraoches taken and suggest ways in which they are likely to influence the statistical behaviour of interest rates. In Section 3, we examine distributional properties of short term interest rates from Germany, the UK and the US employing a range of econometric techniques. In Section 3.2, we study the higher moments of interest rate changes using non-parametric kernel estimates of their distributions. Our results suggest that the degree to which distributions are fat-tailed depends on the monetary control regime. This is especially apparent in the US where two distinct periods of interest-rate- and monetary-targeting are compared. In Sections 3.3 and 3.4, we investigate the persistence of interest rate changes by considering unit root tests and simple autoregressions. The specification of the lag structure in interest rates affects the estimated persistence one obtains in autoregressions in that, in almost all cases we examine, allowing for more lags generates more rapid reversion to the long run mean.

Informed by the kernel estimates and the autoregressions, in Section 4.3, we report the results of Maximum Likelihood estimates of two commonly applied single state variable yield curve models, the Vasicek and the Cox, Ingersoll and Ross models. In each case, we perform the estimation for nine data sets: Sterling, Deutschemark and US dollar Euro-deposit rates of one-, six-, and twelve-month maturities. Our approach to estimation fully allows for temporal aggregation in that the likelihoods we employ are constructed using the conditional densities of the discretely sampled data. We also take account of the fact that the theoretical interest rates implied by the models are affine functions of the underlying instantaneous interest rate, and that the affine mappings involved depend on the parameters of the processes.

The estimates we obtain illustrate the difficulties of fitting short-term interest rate data using standard diffusion models. The Vasicek model results are unstable in that parameters estimated from interest rate data of different maturities (which should be equal) differ significantly. The Cox-Ingersoll-Ross model estimates are more homogeneous across maturities but suffer from the well-known failing of empirical estimates of this model (see Gibbons and Ramaswamy (1993) and Ball and Torous (1995)) that the estimated reversion rates are implausibly large. Again, we interpret this finding as evidence of specification problems. The two models differ only in their specifications of the instantaneous volatility of the short rate. To the extent that these specifications are inaccurate, this is likely to show up in distorted and implausible estimates of 'persistence parameters' like reversion rates which affect volatility over finite periods of time.[2]

In Section 5 of the paper, we exposit a new approach to yield curve modelling, described in more detail in El-Jahel, Lindberg, and Perraudin (1996). Under this approach, short term interest rates are modelled as pure jump processes of which the rate of jump is a function of a diffusion process. Such models, which have been discussed by Babbs and Webber (1994), have the potential to explain the higher moment behaviour of interest rate changes, since the basic assumptions accurately mimic actual practice in most bonds markets, i.e., that the authorities peg a particular key rate at the short end of the yield curve. Moments of simulated bond prices generated from the jump yield curve model are compared to moments of actual bond prices. We show that the characteristic of actual data that kurtosis increases rapidly as data frequency rises is faithfully captured by the model.

[2]The extended Vasicek and Cox-Ingersoll-Ross models discussed in Hull and White (1990) allow the unconditional mean of the short rate to depend upon time. The motivation is that such models permit one to fit the entire structure of discount bond prices at some instant of time. In principle, however, allowing the mean to change could improve the model's ability to mimick the time series properties of short interest rate.

Table 1: MONETARY POLICY AND INTEREST RATES

Characteristics	Germany	United Kingdom	United States
Major instrument	Repo rate	Band 1 stop rate	Adjustment of non-borrowed reserves
Maturity of major instrument	Usually 2 to 8 weeks	1 to 14 days	1 to 14 days
Other instruments	Discount rate and lombard rate	2.30 pm lending rate and minimum lending rate	Discount rate
Operational target rate	Call rate	Base rate	Fed funds rate
Reserve requirements	Yes, averaging over the month	Only clearing balances	Yes, instantaneous reserve accounting
Interest elasticity of reserve demand	High	Low	Low
Fine tuning activity	Low	Very high	High

Sources: Batten, Blackwell, Kim, Nocera, and Ozeki (1990), Freedman (1990) and Kneeshaw and den Bergh (1989).

2 Monetary Policy in Three Countries

2.1 A Schematic Representation

We begin by discussing the monetary control systems of the three countries whose interest rates we shall analyse statistically, Germany, the United Kingdom and the United States. The approaches the three countries take to monetary control differ in many specific aspects but possess important common elements. In general, monetary control systems may be represented with the following simple schema:[3]

instrument	→	operational target	→	intermediate target	→	ultimate goal

Our diagram identifies four distinct elements: (i) the instruments directly controlled by the central bank (its portfolio and the terms of its credit facilities); (ii) the operational target on which the instrument changes operate (usually a very short-term market interest rate); (iii) the intermediate target to which the monetary authorities may commit themselves (money or credit or nominal spending or the exchange rate); and (iv) the ultimate goals that they hope to achieve in the long run. How do our three countries' monetary arrangements fit into this schematic representation?[4] Three important points should be mentioned.

2.2 Interest Rate Instruments and Targets

First, each country has a key interest rate which serves as its operational target. (Summaries of the key interest rates and the ways in which they are manipulated by the autrhorities may be found in Table 1.) One could perhaps argue that base and call rates play this role in the UK and Germany, while the operational target in the US is the rate on Federal funds. In Germany and the UK, the authorities tightly control their operational target rate by adjusting respectively the repo rate (the rate at which the Bundesbank is willing to enter repurchase agreements in government

[3]The terminology we employ here draws on Freedman (1990).

[4]Detailed discussions of implementation procedures in a number of countries can be found in Batten, Blackwell, Kim, Nocera, and Ozeki (1990), Freedman (1990) and Kneeshaw and den Bergh (1989).

bonds with money market participants) and the band 1 stop rate (the rate at which the Bank of England is willing to discount Treasury bills).

Typically, the underlying rates are constant over quite long periods of time and when adjusted move up or down in a discrete jump. The discrete nature of these adjustments means that changes in interest rates further along the yield curve are likely to exhibit leptokurtic behaviour. In fact, if very short interest rates jump by discrete amounts, the shorter the frequency of data one examines, the more fat-tailed the distributions of interest rate changes will appear.

The US system is somewhat different in that the authorities influence their operational target rate, the rate on Federal funds, not by adjusting some other rate but by altering the supply of non-borrowed reserves through open market operations. In consequence, the Fed funds rate shows somewhat more short-term variability than the German and UK operational target rates. Even so, the US authorities adjust non-borrowed reserves with the aim of keeping the Fed funds rate in a narrow corridor around an implicit target rate and the latter tends to adjust discretely when the authorities alter their policy stance.

2.3 Intermediate Targets and Policy Goals

Second, the central banks of the three countries employ different approaches to the use of intermediate targets. The Bundesbank focusses on money growth whereas the Federal reserve and the Bank of England rely on a wide range of leading indicators of the course of inflation and the general economy. However, it is important to distinguish reality from public utterances and realise that adherance to intermediate targets may be more or less strict. For example, the Bundesbank allows large discrepancies to open up between monetary aggregates and its targets and some commentators have argued that one should regard German policy as reacting to the general state of inflationary pressures in the economy.

Third, the three countries considered differ significantly in the way in which they adjust interest rates in the face of inflationary shocks. Such shocks might involve an increase in velocity associated with increases in private or overseas sector demands for domestic goods and services. All three countries broadly-speaking espouse price

Figure 1a. Key rates

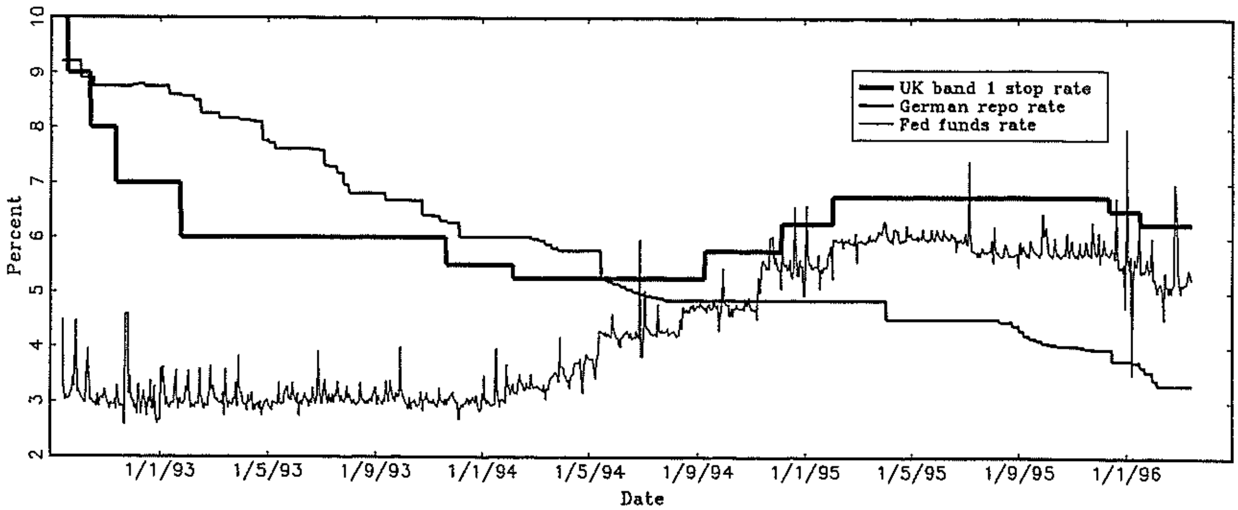


Figure 1b. UK Interest Rates

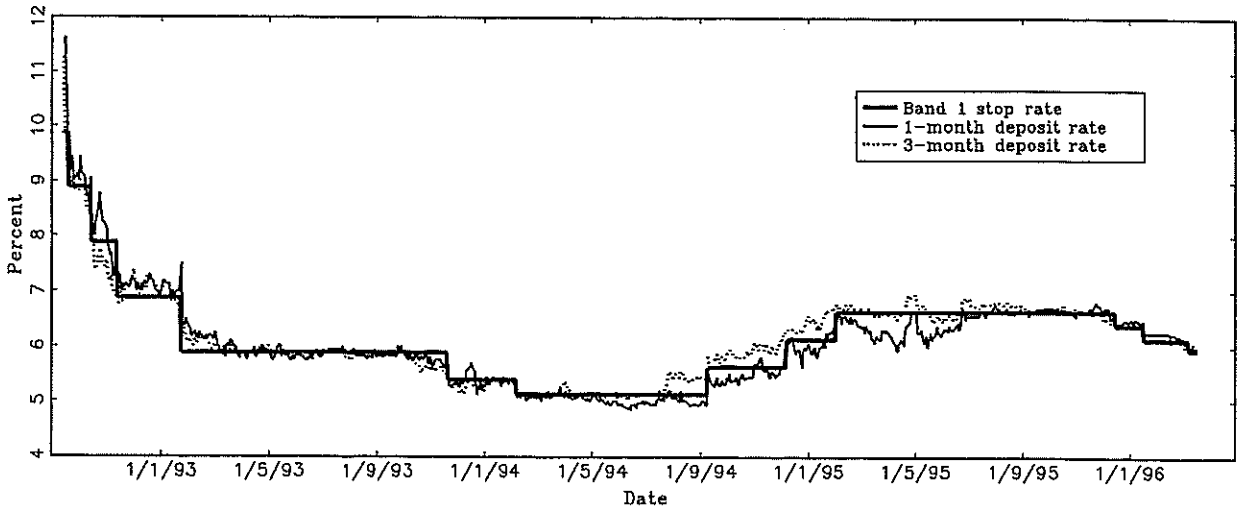
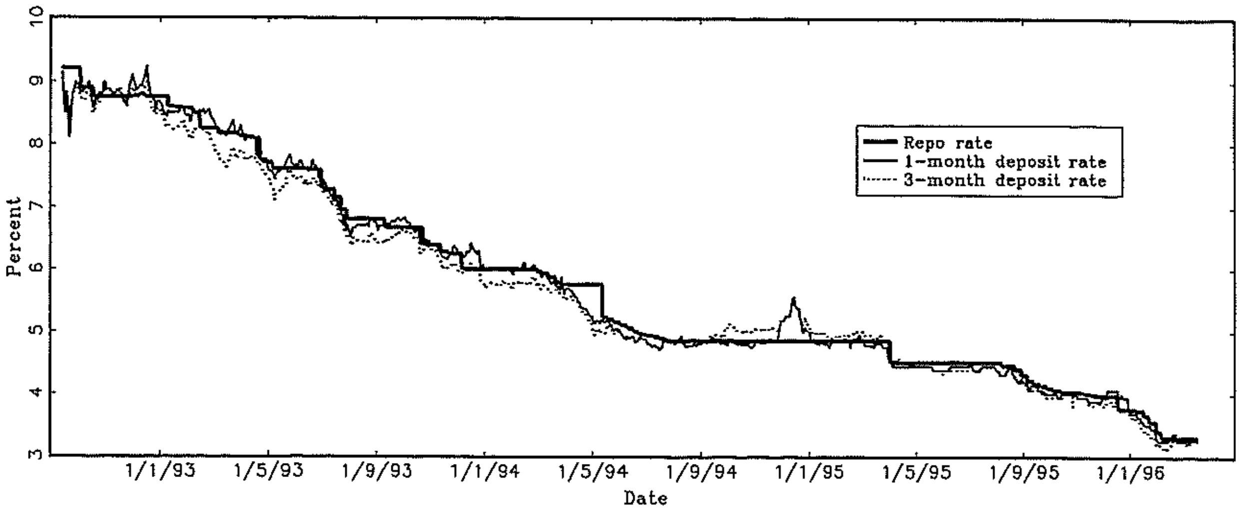


Figure 1c. German Interest Rates



stability as the ultimate goal of monetary policy,[5] but differ in the extent to which they allow this ultimate aim to influence more short term policy. Hard money policies such as that of the Bundesbank may be characterised as reacting firmly to inflation shocks and then resolutely adhering to the higher interest rates adopted. Such behaviour generates slow reversion of interest rates to their long run means. Monetary policy in the UK by contrast typically involves sharp movements in interest rates as inflation picks up but quick downward adjustments in interest rates as soon as this is feasible. Such policies are likely to generate rapid reversion to the mean in interest rates.

3 Interest Rate policies and Distributions

3.1 Key Rates in Three Markets

In Figure 1, We plot daily data on interest rates from three different bond markets we shall study in our empirical work. Those of Germany, the Uk and US [6] In each market, the authorities control a key short term interest rate, thereby aiming to influence the market more generally. In the Appendix, we describe the operation of monetary policy in the above three countries and the role played by various different interest rates. We argue that the key short rates are respectively the repo rate in Germany, the band 1 stop rate in the UK market and the Fed funds rate in the US. In Figure 1a, we show the German repo rate and the UK band 1 stop rate. It is apparent from the figure that these rates are pure jump processes, varying only through sharp, discontinuous movements. The sample paths of the German and UK

[5]The Bundesbank Act of 1957 stipulates that the central bank's fundamental task is to achieve stable prices. Central bank legislation in the United Kingdom and the United States does not explicitly state that monetary policy should aim primarily at on price stability.

[6]In the plots and descriptive statistics we report in earlier sections of the paper, we employ daily data. When we estimate parametric yield curve models (the Vasicek and Cox-Ingersoll-Ross models), it makes more sense to use weekly or lower frequency data since the non-linear, iterative Likelihood Maximization technique we employ performs much better numerically when the data is not too noisy. To facilitate comparisons, we also use weekly data in the unit root tests and autoregressions reported below.

reference rates shown in the Figure possess a number of interesting characteristics.[7]

First, the inter-jump times are highly variable. On occasion, the authorities alter rates through a rapid sequence of moves, while at other times, quite substantial rate changes suddenly occur after a long period of stability. Second, a striking aspect of the sample paths is the very considerable autocorrelation in the signs of interest rate changes, in that there are long periods in which rates move consistently in one direction. Third, step-sizes vary also in a highly dependent way. At times, the authorities have pushed rates gradually up or down through extremely small changes in the level whereas, in other periods, adjustments have been more brusque.

Figure 1a also shows the key US interest rate, the rate on Fed funds. Market-determined to a greater degree than the German or UK key rates, the Fed funds rate nevertheless appears from the plot to move in a reasonably narrow band around a stable, underlying level which changes periodically.[8] Figures 1b and 1c show the behaviour of German and UK 1- and 3-month deposit rates compared to that of the respective key rates. It is interesting to chart the evolution of the market rates around the key rates.

3.2 Leptokurtosis and Official Rates

Now, consider ways in which the distribution of interest rate changes reflects the monetary control arrangements adopted by the authorities. As mentioned in the Introduction, two features of monetary policy are likely to show up in stochastic properties of interest rates (i) the way in which the authorities' control of key rates at the short end of the market will influence higher moments of interest rate changes further down the yield curve, most notably kurtosis and skewness, (ii) the authorities' resolution in adhering to policies will affect the speed of reversion towards the long

[7]The figure shows data from the period since the break-up of the Exchange Rate Mechanism in 1992. The behaviour of rates prior to this date was somewhat different although, broadly-speaking, the distributional features we describe below were present in the earlier period.

[8]Such behaviour, indeed, is consistent with the US authorities' avowed policy of intervening to keep Fed funds in a band around an implicit target level. For example, for most of 1993, the official target level was 3%.

Figure 2a. Standardized 3-Month Rate Distributions

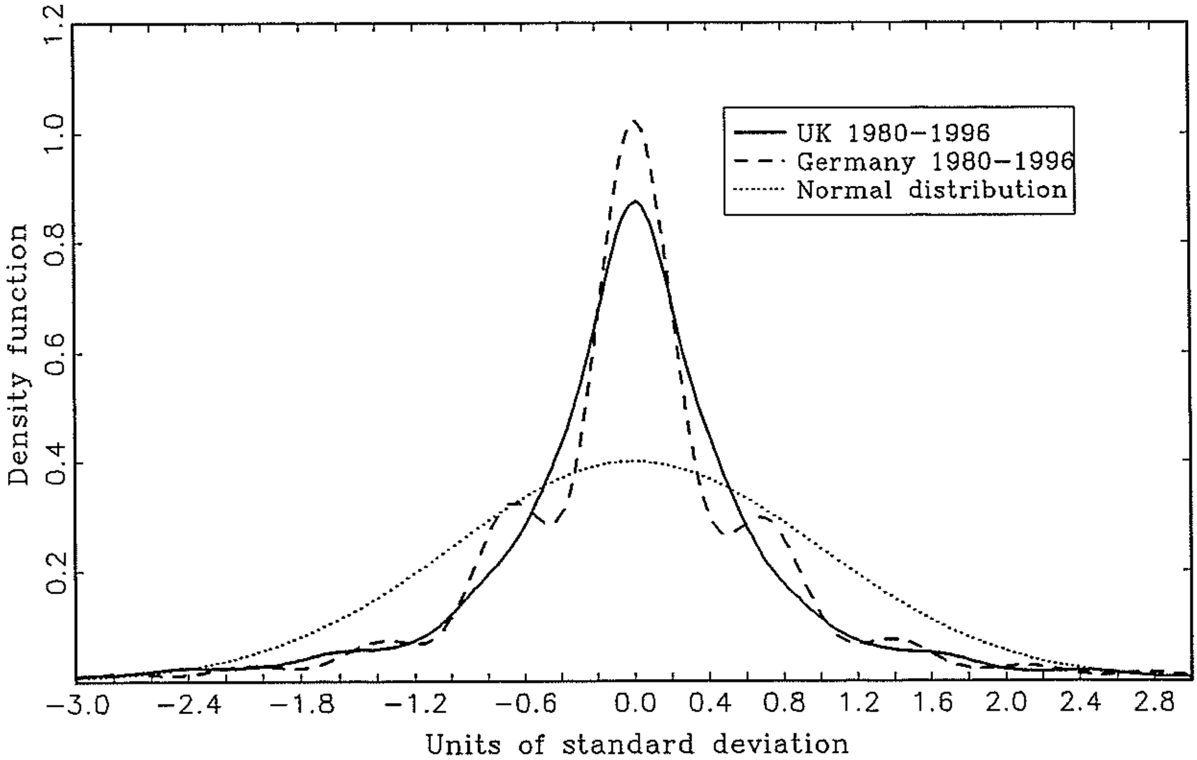
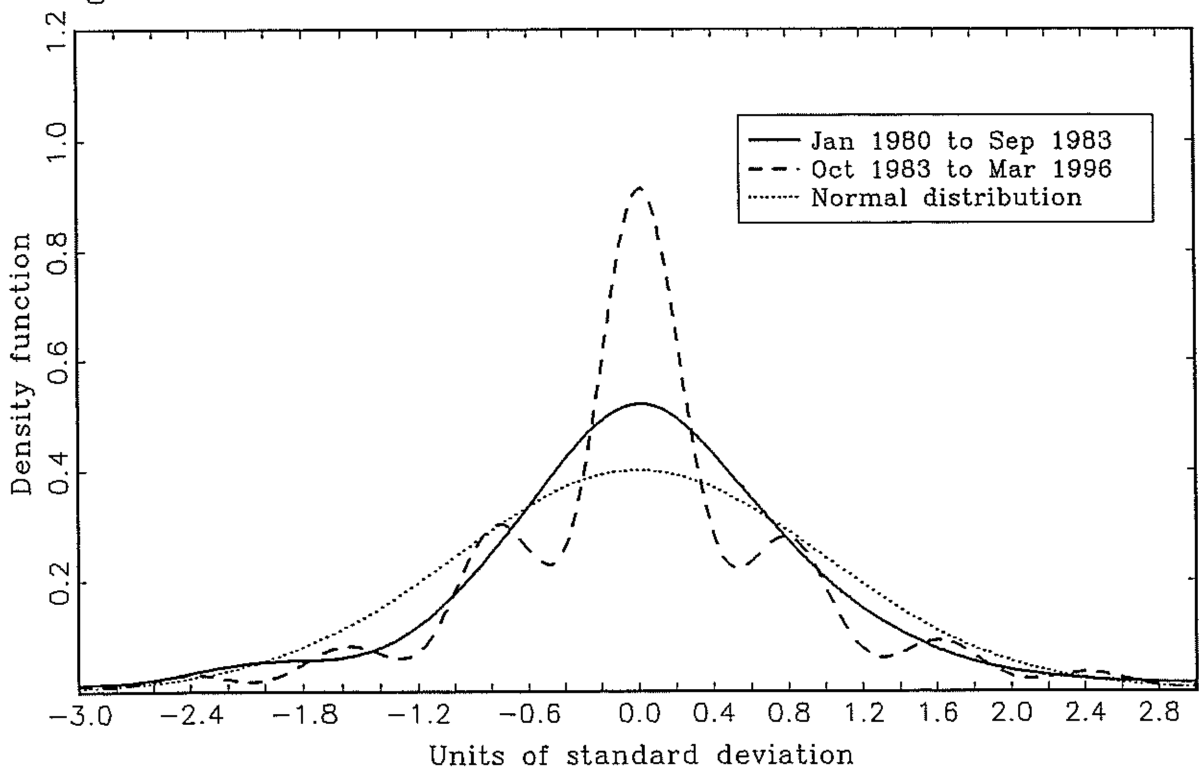


Figure 2b. Standardized US 3-Month Rate Distributions



run mean interest rate.[9]

First, consider the higher moments of interest rate changes. Figure 2a shows estimated densities for daily changes in German and UK 3-month interest rates. The densities depicted are based on non-parametric kernel estimates. The estimates are calculated using a Gaussian kernel and a window size of 1.06×standard deviation/(sample size* (1/5)). For details, see Silverman (1986). For each series, we standardize the data, demeaning and scaling it by the sample standard deviation. This enables us to compare the densities with a standard normal density also shown in Figure 2a.

Evident from the plotted densities is the unconditional leptokurtosis of the interest rate changes. The UK interest rate changes are *less* fat-tailed than those of Germany, in that their respective kurtosis coefficients are 19.2 and 44.7.[10] Recall that the kurtosis coefficient, the ratio of the fourth central moment to the square of the second, is 3 for a normally distributed random variable. It is also interesting to note that the German rate density is tri-modal, reflecting the important influence on the distribution of large jumps in rates. We should stress that this feature of the estimates is not simply an artifact of our kernel estimation techniques. The frequency histogram of the raw data without any smoothing shows a similar trimodal pattern.

Figure 2b shows most clearly perhaps the impact on rate distributions of different monetary arrangements in that the two kernel estimates depicted are the densities of US interest rate changes in two periods, January 1980 to September 1983, and October 1983 to March 1996. The earlier period is unusual in that the Federal Reserve was following a policy of targetting Non-Borrowed Reserves, effectively a measure of base money, and allowing the market to determine the Fed funds rate.[11] Since October, 1983, the Federal Reserve has targeted Borrowed Reserves. In the US system Borrowed Reserves are directly proportional to the Fed funds rate, so this policy has effectively meant targetting interest rates.

[9] Other important feature of distributions that will be affected by monetary policy are the long run average interest rate and the variance of rates. While important, these aspects have been widely discussed in previous studies and are now dwelt on here.

[10] These are calculated using data on raw daily interest changes from January 1980 to March 1996.

[11] Even in this period, it should be noted, the US authorities adjusted their Non-Borrowed Reserve target in part to take account of interest rate developments, so interest rates remained to some extent a target variable.

The density for the earlier period appears somewhat closer to that of the normal distribution also shown in the figure. In fact, the kurtosis for the early period was just 6.3. While this probably represents a statistically significant deviation from the kurtosis of a normally distributed random variable, it is far less than the sample kurtosis of 13.2 that we calculate from the data after October 1983. Finally, the density for US data from the later period exhibits the same kind of tri-modal configuration commented on in the case of Germany above. Once again, sharp and comparatively large interest rates adjustments appear to account for this.

3.3 Interest Rate Persistence

Now consider a second way in which monetary policy affects the distribution of interest rate changes, namely the degree of persistence they exhibit. In this section, we shall examine two different statistical measures of persistence, first, unit root tests, and, second, the rate at which shocks die out as shown by estimated autoregressions.

3.3.1 Unit Root Tests

Unit roots in interest rates imply that there is no tendency for the effects of innovations to die out over time. The presence of a unit root is therefore an extreme form of persistence. Ball and Torous (1995) argue that a problem with recent empirical work on interest rate distributions is that it fails to allow for unit roots in interest rates. To a large extent, the notion that there are unit roots in short-term interest rates is a matter of faith. Standard tests for unit roots have so little power against interesting alternative hypotheses (in particular, that interest rates are stationary but converge somewhat slowly) that no firm conclusion can be reached.

To illustrate this claim, we performed Augmented Dickey-Fuller and Phillips-Peron tests for unit roots on our nine interest rate series. The values of the test statistics are given in Table 2. As one may see, unit roots cannot be rejected for any of our nine interest rates. Rather than immediately concluding that the series is non-stationary, however, one should examine the results of Monte Carlos reported in the lower half of the table. Using interest rate parameters in an economically interesting range, we calculate the proportion of times that one would incorrectly fail to reject a

Table 2: UNIT ROOT TESTS

Interest Rates		ONE LAG MODEL Germany	UK	US
6 month rate	DF	-1.149	-1.629	-1.436
	PP	-1.144	-1.591	-1.420
		FIVE LAG MODEL Germany	UK	US
6 month rate	DF	-1.097	-1.494	-1.474
	PP	-1.099	-1.489	-1.478
		MONTE CARLOS Germany	UK	US
Power	p	13.4%	25.1%	24.9%

NOTES:
DF: Dickey-fuller test and PP: Philipps-Perron test. Critical value at a 5% level is -1.95.
DF and PP with constants give similar results.
Monte Carlos are based on 1000 replications using the estimated parameters from the five-lag autoregressions.
p measures the power of the DF test using the 6 month rate, i.e., one minus the probability of accepting a unit root when rates are stationary.

unit root even when the series is stationary.[12] Repeating the experiment 1,000 times, we found that the relevant percentages were 86.6, 74.9, 75.1.

3.3.2 Autoregressions

Under the assumption that interest rates are stationary, we performed a series of autoregressions for the nine interest rate series. In each case, we ran regressions of interest rate levels on lagged levels using weekly data from 2/1/80 to 13/03/96. In each case, the regression was performed with one and five lags. We report the results in the form of the plots shown in Figure 3. Each plot represents the path followed on average by interest rates according to the estimated model following a one percent positive shock.

The ordering of the degree of persistence across countries is consistent with one's intuitive understanding of the degree to which the three countries in our sample are resolute in their adherance to monetary policies. Shocks to German rates are more persistent than those to UK and US rates, and UK shocks exhibit the least persistence. It is interesting to note, however, that German rates have greater 'long-run' or unconditional variance because of this since the lower degree of reversion is offset by a lower amount of instantaneous volatility.

One way to measure the contrary effects of reversion and instantaneous volatility on unconditional variance is to take the ratio of the log of the AR(1) coefficient to half the variance of the residuals from the interest rate regression. This measure is suggested by the fact that when interest rates follow a continuous time AR(1) process with normal increments (i.e., an Ornstein-Uhlenbeck Process), the unconditional variance of the interest rate is the inverse of this quantity. Hence, we may regard our persistence measure as calculated 'per unit of volatility'. On this basis, Germany does have more rapid reversion in that the measure is 0.133 for Germany and 0.127 and 0.082, respectively, for the UK and US.

[12]The data for the Monte Carlos was generated assuming the interest rate levels are order 5 autoregressions with normally distributed innovations. The autoregression parameters and the variance of the innovations were set equal to the parameter estimates obtained in Section 3.4 below.

3.4 Parametric Yield Curve Models

As the last stage of our empirical investigation of interest rate distributions, we shall implement two standard single state variable yield curve models, those developed by Vasicek (1977) and Cox, Ingersoll, and Ross (1985). These models make simple assumptions about the stochastic process followed by the default-free instantaneous interest rate and the price of interest rate risk and then derive the price of finite maturity bonds consistent with these assumptions.

More specifically, Vasicek (1977) assumes that instantaneous interest rates follow a Gaussian mean-reverting process, the Ornstein-Uhlenbeck process:

$$dr_t = \alpha(\theta - r_t)dt + \sigma dW_t, \tag{1}$$

for constants α, θ, and σ and a standard Brownian motion, W_t. Cox, Ingersoll, and Ross (1985), on the other hand, suppose that instantaneous interest rates satisfy a square-root process:

$$dr_t = \alpha(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t \tag{2}$$

again for constant parameters α, θ, and σ. Discrete time increments in square-root processes of this kind are conditionally distributed as independent non-central chi-squared random variables.

In the Appendix, we provide more details of our empirical implementation of these models. Both models have the convenient feature that interest rates over finite periods of time are affine functions of the instantaneous interest rates where the coefficients in the affine mapping are functions of the parameters α, θ, and σ. This means that is is easy to construct a likelihood for discretely sampled observations of the finite maturity interest rates implied by the two models simply by employing the conditional densities of finite maturity changes in the instantaneous interest rate processes (Gaussian for the Ornstein-Uhlenbeck process and non-central chi-squared for the square-root process), and incorporating a simple Jacobian adjustment term to allow for the fact that the observable is an affine function of the latent variable in question. Note that in our econometric implementation of these models, we suppose that the price of risk is zero. Otherwise, the drift parameters as they enter the affine mapping linking instantaneous with finite maturity interest rates would differ from the corresponding parameters in the distribution of the short interest rate.[13]

We estimate the Vasicek and Cox-Ingersoll-Ross models using interest rate data from our three markets, US dollar, Deutschemark and British pound, and for three maturities, one, six and twelve months. The data consists of Wednesday morning observations on Euro-deposit interest rates over the period 2/1/1980 to 15/03/1996, taken from the Bank for International Settlements database.[14] In all, this yielded 846 observations for each time series. We chose to focus on short-term interest rates rather than using series further along the yield curve since no single-state-variable yield curve model is likely to explain both short and long bond returns very satisfactorily (see the discussions in Litterman and Scheinkman (1991), Gibbons and Ramaswamy (1993), and Pearson and Sun (1994)).

3.5 Parameter Estimates

Table 3 contains our Maximum Likelihood parameter estimates for the Vasicek and Cox-Ingersoll-Ross models. Estimates are reported on an annualized basis and t-statistics appear in parentheses. In estimating these models on finite maturity interest rates, we effectively invert the relationship between the observed interest rates and the implicit instantaneous interest rates assumed by the models. Hence, if either the Vasicek or the Cox-Ingersoll-Ross model were correct, the parameter estimates for that particular model based on one-, six- and twelve-month interest rates should be the same.

The results for the Vasicek model are notable for the instability of parameters estimated from interest rate data of different maturities. The reversion parameter for Deutschemark interest rates is 1.2, 0.7 and 0.6 depending on which data one employs. It is also noticeable that the unconditional mean interest rates according to the model, the θ parameters, differ significantly from the actual unconditional mean of interest rates over the period covered by the sample. Both of these features of

[13] An alternative might be to infer the value of the risk adjustment from the price of some yield-curve-related asset. Attempting to estimate the risk adjustment simply using the time series data we here employ is not feasible since it is scarcely identified.

[14] This data consists of quotes fixed at 11am Swiss time. It is preferable to use Wednesday observations since relatively few public holidays fall on this day.

the estimates is suggestive of misspecification. This, after all, would not be surprising given that Ornstein-Uhlenbeck processes have normally distributed discrete time increments (implying kurtosis of 3) yet the sample kurtosis coefficients of the data for changes in the Deutschemark, sterling and US dollar one-month interest rates respectively are 15.0, 8.8, and 19.2.

On the other hand, the results for the Cox-Ingersoll-Ross model reported in the lower half of Table 4 exhibit remarkable homogeneity in parameter estimates for different interest rate maturities. Also, the unconditional mean interest rates, θ, are quite close to the corresponding sample means.[15] Both these aspects of the parameter estimates suggest that the square root process may provide a good model of the data. However, the parameter estimates of the square root process we report suffer from the failing noted in past empirical studies of the Cox-Ingersoll-Ross model such as Gibbons and Ramaswamy (1993) and Ball and Torous (1995), namely that the estimated reversion rates, the α parameters, are implausibly large.

In Table 3, the reversion rates range from 34 to 52. Recall that the parameters are reported on an annualized basis so the results imply that the interest rate should revert to its unconditional mean within a couple of weeks of a shock. These rates of reversion may be compared with the far lower reversion rates obtained in the autoregressions reported in Section 3.3.2.

Ball and Torous (1995) argue that estimates of autoregression parameters may be biased up in small samples if the series in question is close to a unit root. However, the magnitude of the biases that Ball and Torous generate in Monte Carlo simulations is too small to account for the very rapid mean reversion rates obtained by Gibbons and Ramaswamy (1993) and others including ourselves. Also, such biases would affect estimates not just of the continuous time models investigated in this section but also the autoregression models of Section 3.3.2, whereas the latter yielded quite low reversion rates.

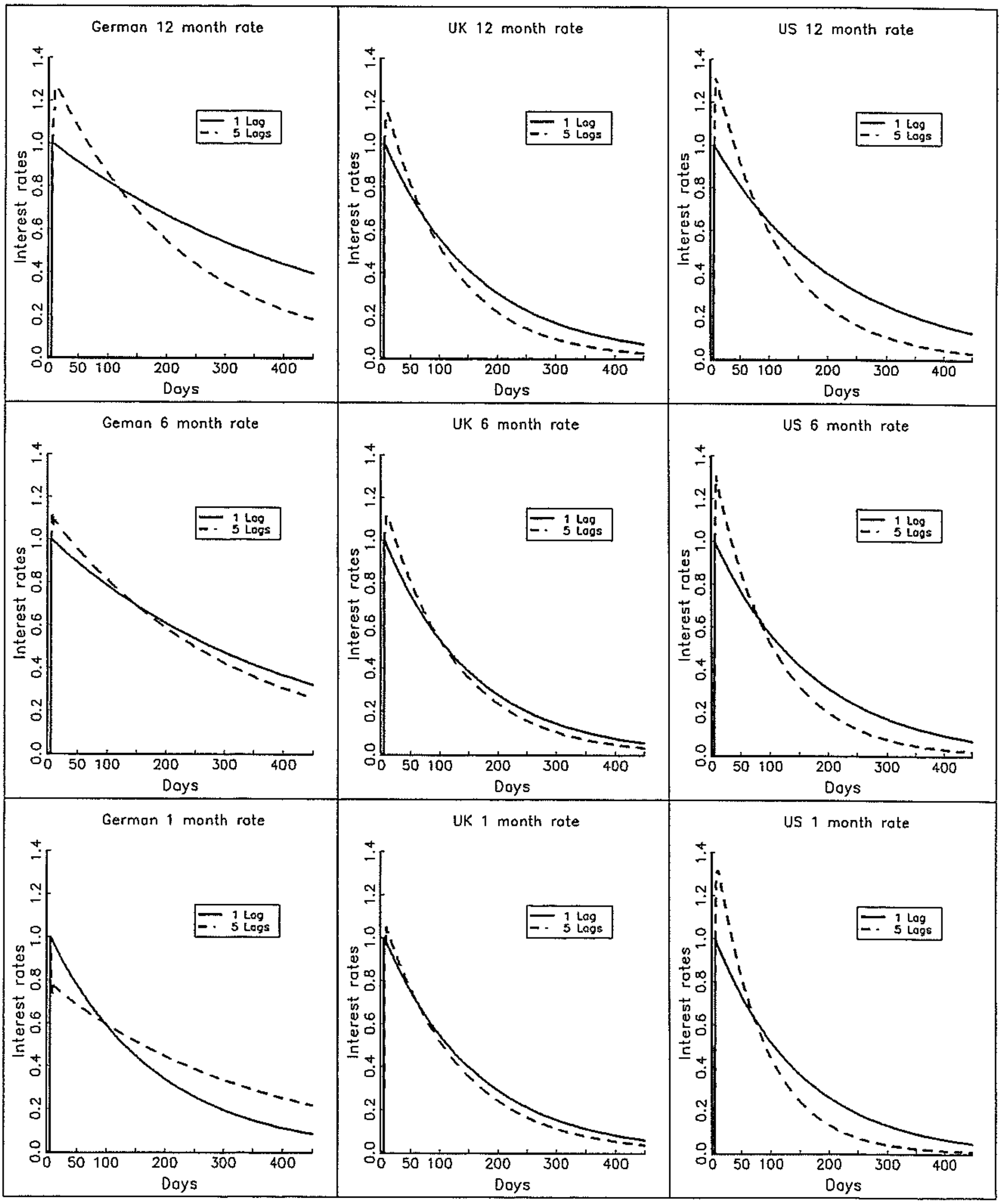
[15]One may compare the estimates of θ in the table with the sample means of the one, six and twelve month rates. For Deutschemark rates, these were: 6.7%, 6.8% and 6.8%. For sterling rates, they were 10.7%, 10.7% and 10.6%. For US dollar rates, they were 8.2%, 8.5%, and 8.6%.

Table 3: YIELD CURVE MODEL ESTMATES

		VASICEK MODEL					
		Germany		UK		US	
One	α	1.248	(1.447)	1.225	(1.721)	1.662	(1.943)
month	θ	0.057	(3.299)	0.086	(3.809)	0.069	(3.395)
rates	σ	0.040	(20.163)	0.046	(20.266)	0.064	(20.163)
Six	α	0.723	(1.052)	1.561	(1.899)	1.336	(1.740)
month	θ	0.050	(1.805)	0.090	(4.871)	0.068	(2.900)
rates	σ	0.032	(20.280)	0.050	(20.188)	0.057	(20.224)
Twelve	α	0.631	(0.888)	1.359	(1.660)	1.084	(1.473)
month	θ	0.050	(1.548)	0.089	(4.473)	0.069	(2.584)
rates	σ	0.030	(20.258)	0.047	(20.195)	0.051	(20.247)
		COX-INGERSOLL-ROSS MODEL					
		Germany		UK		US	
One	α	44.670	(11.019)	46.251	(22.791)	33.990	(17.894)
month	θ	0.067	(27.890)	0.107	(30.355)	0.082	(18.823)
rates	σ	0.839	(38.558)	1.008	(39.495)	1.051	(38.302)
Six	α	46.316	(11.289)	48.950	(11.817)	35.034	(9.230)
month	θ	0.068	(29.316)	0.107	(32.618)	0.084	(19.749)
rates	σ	0.833	(38.626)	0.990	(38.883)	1.047	(39.309)
Twelve	α	48.299	(11.596)	52.150	(12.271)	36.914	(9.672)
month	θ	0.068	(30.970)	0.106	(35.568)	0.086	(21.531)
rates	σ	0.823	(38.340)	0.964	(38.418)	1.022	(39.720)

NOTES: All parameters are reported on an annualised basis.
T-statistics appear in paretheses after the parameter.
Maximum Likelihood estimation was performed using weekly data on discount bond yields from 2/1/80 to 13/3/96.
The CIR model assumes the short rate, r_t, follows:
$dr_t = \alpha(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t.$
The Vasicek model assumes r_t follows:
$dr_t = \alpha(\theta - r_t)dt + \sigma dW_t.$

Figure 3: Interest Rate Persistence



3.6 Specification Problems

The obvious difference between the continuous time models described in Sections 3.4 and 3.5 and the autoregressions of Section 3.3.2 is that the former impose restrictions across the conditional means and higher moments of interest rate changes while the latter impose no restrictions on the second or higher moments. To see the potential significance of this, consider the way in which parameters α and σ interact in diffusion models of the kind we are investigating. Conditional means are relatively ill-determined statistically in data sets of interest rate changes so the estimates of the parameter α will be most strongly affected by its contribution to the variance and higher moments.

A given variance in discretely-sampled data on interest rate changes may be matched by different combinations of α and σ since increasing the two parameters together generates greater instantaneous volatility but more rapid reversion within the finite period in question. One may then fit higher moments such as kurtosis as well as a given variance by selecting some combination of α and σ values that can mimic both statistical properties simultaneously. Leptokurtic behaviour will be captured in such models by high instantaneous volatility combined with high reversion rates. Such a combination of values will generate substantial interest rate shocks without boosting variance.

If the model were correctly specified, then it would not matter that a parameter like α which influences both the conditional means and the higher population moments of interest rate changes be largely determined by the higher sample moments. However, if the model is misspecified, as it undoubtedly is, the estimates of the mean that one obtains will be highly distorted. Unfortunately, the mean is crucial for most pricing applications and a model that cannot fit the evolution of the conditional mean is therefore unlikely to be of little use.

4 A Jump Model of Interest Rates

4.1 Basic Assumptions

In this section, we suggest a rather different approach to modelling interest rates and the term structure than that followed in the classic diffusion models of Vasicek (1977) and Cox, Ingersoll, and Ross (1985). A general version of the approach we describe has been discussed by Babbs and Webber (1994) while El-Jahel, Lindberg, and Perraudin (1996) obtain analytical solutions under specific distributional assumptions. The model of this section is taken from El-Jahel, Lindberg, and Perraudin (1996).[16]

The basic approach consists of assuming that short-term interest rates are a pure jump process, pegged by the monetary authorities and periodically adjusted by discrete amounts. Of course, bond prices further along the yield curve diffuse as well as jump so there is need to introduce a state variable that can be regarded as summing up information about future government interest rate policies. We therefore assume that the rate of jump, γ_t, for the government-controlled short interest rate, r_t, is a function of the current level level of a diffusion process, X_t,

$$\frac{d}{d\Delta}\mathrm{E}_t\left(r_{t+\Delta}\right)\bigg|_{\Delta\downarrow 0} = \delta_t\gamma(X_t). \tag{3}$$

where $\gamma(X_t)$ is the rate of jump or jump intensity, and δ_t is the expected jump size of a jump in the instantaneous interest rate at t should one occur (where the expectation is conditional on information at $t-$). We further suppose:

1. that the diffusion state variable, X_t, is an Ornstein-Uhlenbeck process:

$$dX_t = \alpha(\theta - X_t)dt + \sigma dW_t, \tag{4}$$

2. that the jump rate function, $\gamma(.)$ is quadratic:

$$\gamma(X_t) = \beta X_t^2, \tag{5}$$

[16]One should perhaps note that this way of introducing jumps into interest rates differs significantly from earlier work such as Ahn and Thompson (1988) and Shirakawa (1991) which incorporated independent and identically distributed jump components in jump-diffusion models of short-term interest rates and the yield curve.

3. that future jump sizes are a sequence of known constants, $\delta_1, \delta_2, \delta_3 \ldots$,

4. and that agents are risk neutral.

The assumption of risk neutrality is made here for expositional convenience. One may introduce risk aversion using standard change of measure arguments. The assumption of fixed and known jump sizes is also made for simplicity. There are various ways to relax this and introduce stochastically varying jump sizes. El-Jahel, Lindberg, and Perraudin (1996) develop a model in which the jump size at t (should a jump occur) equals the level of an arithmetic Brownian motion, independent of the Ornstein-Uhlenbeck process driving the jump rate function.

4.2 Applying the Karhunen-Loeve Theorem

These assumptions fully specify the stochastic behaviour of short-term interest rates. To derive the price $P_{t,T}$ of a discount bond maturing at date T, we need to evaluate the expectation:

$$P_{t,T} = \mathrm{E}_t \left(\exp \left[- \int_t^T r_s ds \right] \right). \tag{6}$$

At any time, t, the level of interest rates, r_t, may be written as:

$$r_t = r_0 + \sum_{j=1}^{N(t)} \delta_j \tag{7}$$

where $N(t)$ is the number of jumps up to and including time t. The integral in Eq. 6, can then be written:

$$\int_0^T r_s ds = r_0 t_1 + (r_0 + \delta_1)(t_2 - t_1) + (r_0 + \delta_1 + \delta_2)(t_3 - t_2) + \ldots + \left(r_0 + \sum_{j=1}^{N(T)} \delta_j \right) (T - t_{N(T)}) \tag{8}$$

Cancelling terms, one may then write Eq. 6 as:

$$P_{t,T} = E_t \left[\exp \left(-r_0 T - \sum_{j=1}^{N(T)} \delta_j (T - t_j) \right) \right]. \tag{9}$$

El-Jahel, Lindberg, and Perraudin (1996) show how one may calculate for the simple case in which the $delta_i$ are known constants and for another model in which they are proportional to the level of an independent Brownian motion. To give a flavour of their analysis, we shall describe the case in which the δ_i are known constants.

The first step is to solve for the density of the sample paths of the jump process (equivalent here to deriving the joint density of the jump times conditional on information at t). Let $\rho\,[N_t : 0 \leq \tau < 1]$ denote the path density of the counting process that records changes in interest rates. Here, we have chosen time units so that the horizon $[t, T]$ is normalized to $[0, 1]$. Conditional on the time path followed by the forcing process, X_t, the counting process, N_t for interest rate changes is a Poisson process. By Snyder and Miller (1991) page 358, it follows that:

$$\rho\,[N_t : 0 \leq \tau < 1] = E_0\left[\exp\left(-\int_0^1 \gamma(\tau, X_\tau)d\tau + \int_0^1 \ln\gamma(\tau, X_\tau)dN_\tau\right)\right]\,. \qquad (10)$$

To evaluate the expectation in equation (10), El-Jahel, Lindberg, and Perraudin (1996) use techniques employed in the physics literature in models of photon emission (see Macchi (1971)). The main step in this is to express the conditional distribution of the Ornstein-Uhlenbeck forcing process, X_t, as an expansion in terms of eigenfunctions, using the Karhunen-Loeve Theorem. In particular, El-Jahel, Lindberg, and Perraudin (1996) show that:

Proposition 1 *By the Karhunen-Loeve Theorem, X_t can be written as:*

$$X_t = \sum_{n=1}^{\infty} x_n \phi_n(t) \qquad \text{for} \qquad t \in [0, 1] \qquad (11)$$

where the $\phi_n(t)$ for $t \in [0, 1]$ are functions of the form:

$$\phi_n(\tau) = \frac{2\sin(\omega_n \tau)}{\sqrt{2 - \frac{\sin(2\omega_n)}{\omega_n}}} \qquad (12)$$

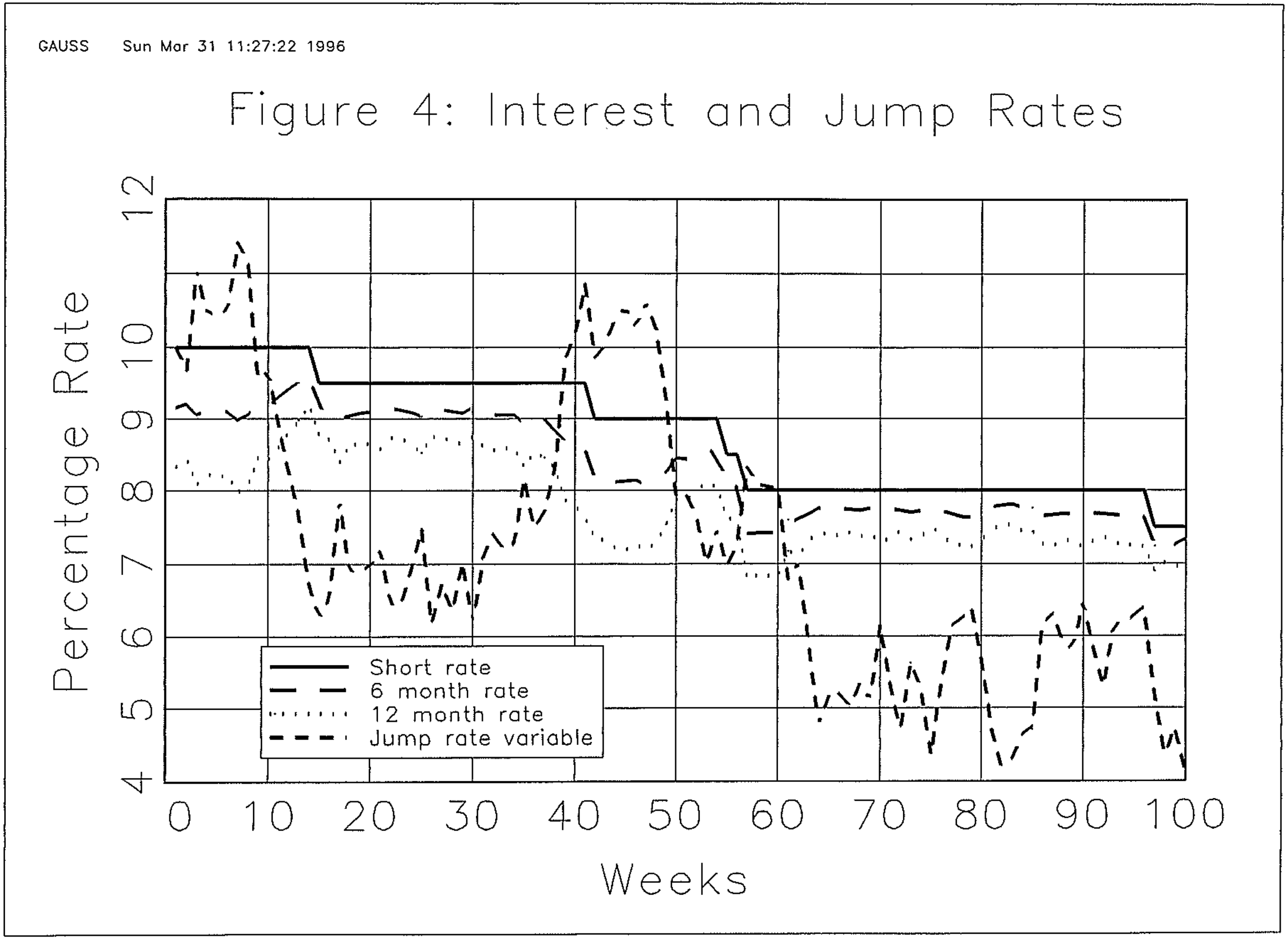
Here, the $\omega_1, \omega_2, \omega_3 \ldots$ are the positive roots of the equation,

$$\frac{2\alpha i\omega_n}{\omega_n^2 + \alpha^2} = \alpha\left(\frac{\exp[-\alpha + i\omega_n] - 1}{-\alpha + i\omega_n} - \frac{\exp[-\alpha - i\omega_n] - 1}{-\alpha - i\omega_n}\right) \qquad (13)$$

The x_n are real-valued, normally distributed independent random variables with variances, $v_n^2 = \sigma^2/(\omega_n^2 + \alpha^2)$ while the means, $m_n \equiv \mathrm{E}_0 x_n$, are given by:

$$m_n = \frac{\eta_n(X_0 - \theta)}{\omega_n^2 + \alpha^2}\left\{\omega_n - \exp[-\alpha](\alpha\sin(\omega_n) + \omega_n\cos(\omega_n)\right\} + \frac{\eta_n\theta}{\omega_n}(1 - \cos(\omega_n)) \quad (14)$$

where $\eta_n \equiv 2/\sqrt{2 - \sin(2\omega_n)/\omega_n}$.


GAUSS Sun Mar 31 11:27:22 1996
Figure 4: Interest and Jump Rates
Percentage Rate
12
10
9
8
7
6
5
4
0
10
20
30
40
50
60
70
80
90
100
Weeks
Short rate
6 month rate
12 month rate
Jump rate variable

Proof: see El-Jahel, Lindberg, and Perraudin (1996). □

As El-Jahel, Lindberg, and Perraudin (1996) discuss, in any practical application, the infinite sum in equation (11) must be truncated at some finite number N of terms in the summation. In fact, rather than using the sequence of eigenfunctions, ϕ_1, ϕ_2, ϕ_3, ..., ϕ_N, it turns out that it is better in practice to employ as eigenfunctions: ϕ_0, ϕ_1, $\phi_2, \phi_3, \ldots, \phi_{N-1}$, where ϕ_0 is defined as the orthogonal projection of a constant onto the ϕ_i, $i = 1, 2, \ldots, N-1$.

4.3 The Interest Rate Path Density

The orthogonality properties of the Karhunen-Loeve representation enable one to write the expectation in equation (10) in the following simple form:

$$\rho\left[N_\tau : 0 \leq \tau < 1\right] = E_0\left[\exp\left[-\beta\sum_{s=1}^{\infty} x_s^2 + \int_0^1 \ln(\beta X_\tau^2) dN_\tau\right]\right] \tag{15}$$

where N_τ is the counting process associated with the Poisson process. It is much easier to evaluate this and hence obtain the sample path density of interest rates than it is to attempt a more direct evaluation of the expression in (10). In fact, the path density ρ is:

Proposition 2 *If the jump times are denoted t_1, t_2, ..., t_L,*

$$\rho[N_\tau : 0 \leq \tau < 1] = \sum_{L=0}^{\infty} \beta^L \sum_{\substack{j_1, \ldots j_L = 1 \\ k_1, \ldots k_L = 1}}^{\infty} a_L(j,k) \prod_{l=1}^{L} \phi_{j_l}(t_l)\phi_{k_l}(t_l) \tag{16}$$

where:

$$a_L(j,k) = \prod_{n=1}^{\infty} E_0\left[x_n^{p_n(j)} x_n^{q_n(k)} \exp\left[-\beta x_n^2\right]\right]. \tag{17}$$

The vectors $(j_1, \ldots j_L)$ and $(k_1, \ldots k_L)$ are L-dimensional permutations of the positive integers. The inner summation in equation (16) is thus over all these permutations. $p_n = p_n(j)$ and $q_n = q_n(k)$ are defined as the number of elements in permutations j and k that equal a given integer, n. [17]

[17]The evaluation of the $a_L(j,k)$ is described in detail in El-Jahel, Lindberg, and Perraudin (1996).

Proof: see El-Jahel, Lindberg, and Perraudin (1996). □

Once one has the path density of the interest rate process, it is simple to obtain the bond price by evaluating the expectation in equation (6). Further details are spelt out in El-Jahel, Lindberg, and Perraudin (1996).

4.4 Bond Yields and Sample Paths

Figure 4 shows a typical set of sample paths for the state variable, X_t, the instantaneous interest rate, r_t and the yield to maturity, $R_{t,T}$ on discount bonds with six and twelve month maturities. The solutions are calculated assuming that the size of interest rates changes is deterministic and given by $\delta = -0.5\%$. Of course, if we were seeking to model anything except short term bonds, this assumption would be very restrictive. Over a horizon of six months or a year, however, it seems reasonable to suppose that market participants belief that rates can only move in one direction and that the jump size is some 'conventional' amount like half a percentage point. The other parameters used in calculating the interest rates in Figure 4 are: $\alpha = 0.5\sqrt{700}$, $\theta = 0.05$, $\sigma = 0.05\sqrt{700}$, $r_0 = 0.1$, and $X_0 = 0.1$. We also assume that the rate of jump equals $\beta \times X_t^2$ where $\beta = 700$.

The sample path shown may be interpreted as follows. The jump rate starts at its unconditional mean, 0.1. This produces a steeply downward sloping yield curve since downward movements in interest rates are anticipated. After 15 weeks, the state variable, X_t, falls markedly, leading to a partial flattening of the yield curve and a long period in which rates do not decline. After a year, the chance of a rate decline rises again. Several interest rate alterations do actually occur and again the yield curve becomes steep. Towards the end of the sample period, the state variable, X_t, remains low and the yield curve becomes less downward-sloping.

4.5 Higher Moments

In Table 4, we compare the higher moments of discount bond yields implied by a typical sample path from our model with those of the interest rate data used in Sections 3.4 and 3.5. Again, for simplicity, we take the known jump size version of the El-Jahel, Lindberg, and Perraudin (1996) model. The moments are calculated

Table 4: INTEREST RATE MOMENTS

Frequency (weeks)	£	DM	$	Δr	ΔR
VARIANCE					
1	0.07	0.04	0.06	0.02	0.02
2	0.16	0.07	0.12	0.04	0.04
4	0.35	0.10	0.16	0.10	0.11
8	0.72	0.18	0.33	0.22	0.23
13	0.89	0.24	0.39	0.40	0.40
KURTOSIS					
1	8.82	14.99	19.23	14.80	17.75
2	5.40	11.90	14.87	14.42	12.57
4	5.09	6.39	5.35	7.82	7.74
8	2.95	3.73	4.17	5.68	5.43
13	2.18	2.77	2.19	3.61	3.77
SKEWNESS					
1	1.16	0.69	0.50	-0.94	-0.69
2	0.61	-0.21	0.09	-0.84	-0.82
4	0.63	0.80	0.43	-0.29	-0.32
8	0.21	0.53	0.00	0.67	0.55
13	0.25	-0.53	0.28	0.11	0.15

NOTES: Columns 2, 3, and 4 contain moments of changes in Sterling 3 month (£), Deutschemark 3 month (DM), and US dollar 3 month ($) interest rates.
Columns 5 and 6 contain moments of changes in the instantaneous and 3 month interest rates implied by the model with parameters:
$\alpha = 0.5$, $\theta = 0.1$, $\sigma = 0.05$,
$\delta = -0.005$ or 0.005, $\beta = 250$.
Rates are expressed in percent.

for returns covering increasing periods of time. As one may see from the table, a characteristic of the data is a steep rise in kurtosis as the frequency of the data shortens. Over quarterly periods, actual interest rate changes have kurtosis of between 3 and 6, whereas weekly changes in interest rates exhibit kurtosis coefficients of 8.8, 15.0, 19.2, and 14.8.[18] The kurtosis coeffients implied by the interest rate sample path generated by our model show a similar steep rise as the data frequency is reduced.

5 Conclusion

In this paper, we have investigated the distributions of short-term interest rates denominated in three different currencies, and stressed their dependence on the monetary control regimes operated by the corresponding national monetary authorities. Countries which peg the short interest rate, periodically adjusting it in discrete steps, tend to possess market rates further along the yield curve which have strikingly high kurtosis. Comparison of US interest rate data for periods of money and interest rate targetting provides further evidence that this is the case.

A second distributional property we examine is the speed with which short-term interest rates in different currencies converge to their long run averages. Again, we relate this to aspects of the approach taken to monetary policy, in this case to the degree of resolution shown by the national monetary authority in the face of a monetary shock. To illustrate the importance of the distributional properties we examine, we estimate the parameters of two standard yield curve models, namely the Vasicek and Cox-Ingersoll-Ross models. We discuss the relationship between their empirical failings and our prior examination of leptokurtosis and interest rate reversion rates.

In particular, we argue that the well-known tendency for estimates of reversion rates in the Cox-Ingersoll-Ross to be implausibly high reflects the fact that it is difficult to mimic the characteristic high kurtosis in high frequency short term interest rate data with standard diffusion models of the instantaneous interest rate. As an alternative approach, in the last section of the paper, we describe a new approach

[18]These differ from the kurtosis coefficients reported in Section 3.2 because the latter are based on daily interest rate changes.

to yield curve modelling developed by El-Jahel, Lindberg, and Perraudin (1996) in which the instantaneous interest rate is taken to be a pure jump process of which the rate is fixed and occasionally changed in discrete steps by the monetary authorities.

6 Appendix

6.1 Monetary Policy in Three Countries

We here describe the operation of monetary control and its impact on money markets in Germany, the UK and the US. In particular, we focus on the elasticities of money market supply and demand and their dependence on the institutional arrangements.

6.1.1 Germany

Figure 5 illustrates the German system of monetary policy instruments. Banks are able to borrow a limited amount of reserves at the relatively low discount rate (i_0 in the figure) and unlimited amounts at the higher lombard rate (i_1). The two rates provide lower and upper bounds for the range within which the call rate (the overnight rate in the interbank market) fluctuates.

The high elasticity of demand for reserves in the German system is largely a consequence of the fact that bank reserve requirements are based on monthly averages. The averaging implies that unexpected temporary shifts in either the demand or supply of reserves will affect the overnight rate relatively little since any excess or shortage of reserves can be made up in the course of the month.

In the past decade, open market operations involving repurchase agreements or repos have been the most important monetary policy instrument of the Bundesbank both for controlling the supply of reserves and adjustments in interest rates. The Bundesbank uses the repo rate to guide the call rate below the lombard rate. Hence, one may view the call rate as an operational target of the Bundesbank.

Until September 1992, repos were usually of one or two months maturity. But, in an effort to gain more operating flexibility, the Bundesbank now alos employs two week repos. Two different kinds of repos are employed: fixed-rate repos with volume tenders and variable-rate repos with interest rate tenders. Interest-rate tenders have been more common but the Bundesbank occasionally prefers volume tenders, especially if money market rates or liquidity allocation has deviated significantly from the levels desired by the authorities. Volume tenders allow the Bundesbank to signal preferred money market rates. Repo bidding usually occurs every Tuesday and

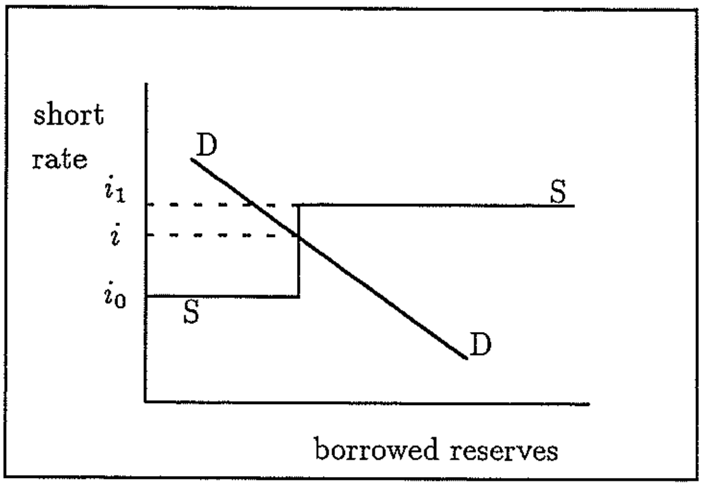


Figure 5: German Monetary Policy

allotments are announced around 10.00 a.m. the following day.

In addition to securities repurchase agreements, the Bundesbank employs a number of so-called "reversible fine-tuning" measures to adjust the supply of reserves on a day-to-day basis, thereby avoiding excessive fluctuations in short-term interest rates. These include outright transactions in short-term treasury bills, repo operations with maturities down to two days and swapped foreign exchange transactions.

6.1.2 United Kingdom

The British system differs significantly from that of Germany. First, the ultimate authority for interest rate changes lies with the Chancellor of the Exchequer and the role of the Bank of England is limited to advising on policy formation and implementing policy changes. Second, so-called discount houses occupy an important position between commercial banks and the Bank of England. The discount houses are private companies whose role is to intermediate surpluses or shortages of funds by discounting treasury or commercial bills. In normal day-to-day operations, the Bank of England's only counterparties are the discount houses. Lending to discount houses is usually at maturities from overnight to fourteen days. The UK does not

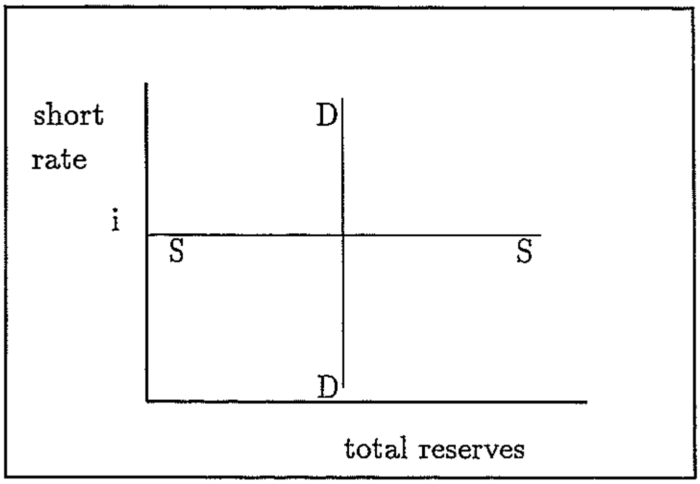


Figure 6: United Kingdom Monetary Policy

operate reserve requirements, but banks hold a small amount of clearing balances (represented in Figure 6 by the vertical DD curve).

In principle, the Bank of England determines rates by selling or buying securities at terms of its choosing. The supply curve of reserves is therefore horizontal. There are two principal ways in which changes in monetary policy are signalled. First, by changes in the stop rate for Band 1 bills which is the lowest rate of discount at which the Bank is prepared to buy bills with maturities of up to 14 days from discount houses. A significant fall or rise in the stop rate is associated with a corresponding movement in the base lending rate of commercial banks.[19] Second, on some occasions the Bank does not provide sufficient funds to the market at the rates on bills offered by the discount houses.

The discount houses are then invited to borrow at 2.30 p.m. at the 2.30 p.m. lending rate. If this rate differs significantly from the most recent dealing rates on eligible bills, a clear signal is given to the commercial banks and a change in their base lending rates generally follows. An even stronger signal is given if the Bank

[19]Base rate are administered interest rates for commercial loans, in some respect analogous to prime rates in the US.

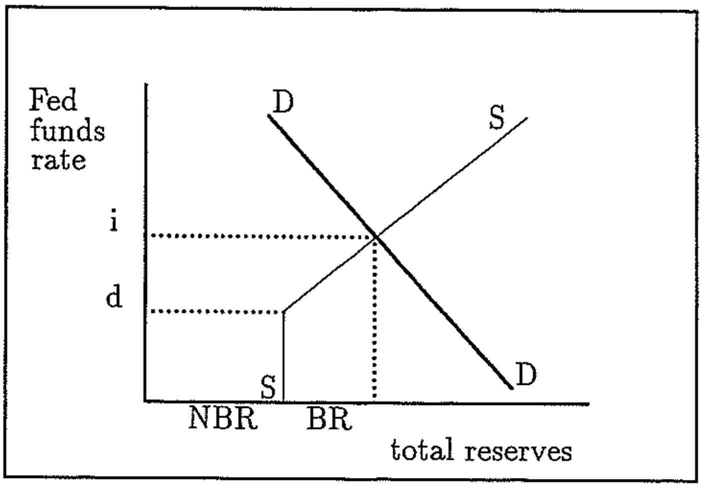


Figure 7: United States Monetary Policy

announces a fixed minimum lending rate for all its dealing with discount houses. In principle, any rate at which the Bank of England deals could be viewed as a key rate, but de facto the base lending rate of banks, which is largely determined by the Bank, plays this role.

6.1.3 United States

The nominal operational target is the level of borrowings at the Federal Reserve's discount window, which is associated with a desired path of non-borrowed reserves. However, the actual operational target is the level of the Fed funds rate, which is the interest rate in the overnight interbank market on deposits at the Fed that depository institutions with reserve deficiencies borrow from institutions with excess reserves. The demand for reserves is fairly interest inelastic being the sum of required reserves, which are very inelastic, and excess reserves which are slightly elastic.

A key factor in the US procedures is the nature of commercial banks' access to borrowed reserves (BR), that is, discount window borrowing. Such reserves are provided at the discount rate but are rationed by virtue of the administrative guidelines on borrowing imposed by the Federal reserve . From the point of view of the banks, there is a trade off between the pecuniary gains and the non-pecuniary costs of borrowing. Hence, the higher the federal funds rate relative to the administrative discount rate, the more banks are willing to borrow at the Fed.

The Fed influences the Fed funds rate (i) through the discount rate and (ii) by adjusting the supply of non-borrowed reserves (NBR). The primary monetary instrument is open market operations in which the Fed buys or sells securities, thereby adding or draining non-borrowed reserves from, the banking system. The Fed rarely performs outright purchases or sales. More frequently, it uses repurchase or matched sale agreements. These agreements typically last for only 1 day, rarely for more than 7 days, and never exceed 15 days. The discount rate is of secondary importance, but is the floor for the Fed funds rate and is the only rate that the Fed formally admits to controlling. Thus, when it changes the discount rate, the Fed is making a particularly strong statement about the direction in which it seeks to move the Fed funds rate.

6.2 Estimation of Parametric Yield Curve Models

In this Appendix, we sketch the approach we took in estimating the parameters of the parametric yield curve models of Section 3.4. Since the transition densities of the state variable in both the Vasicek and the Cox-Ingersoll-Ross models is known, it is straightforward to derive a Maximum Likelihood estimator. The conditional densities of the two interest rate processes, $\rho(r_t|r_{t-1})$, may be summarized as:

1. Vasicek Model (normal density)

$$\rho(r_t|r_{t-1}) = \frac{1}{\sqrt{2\pi}v}\exp\left[-\frac{1}{2}\frac{(r_t - r_{t-1} - m_t)^2}{v^2}\right] \tag{18}$$

$$m_t \equiv (\exp[-\alpha] - 1)(r_{t-1} - \theta) \tag{19}$$

$$v^2 \equiv \frac{\sigma^2}{2\alpha}(1 - \exp[-2\alpha]) \tag{20}$$

2. Cox-Ingersoll-Ross Model (non-central Chi-squared density)

$$\rho(r_t|r_{t-1}) = c\exp[-d_t - e_t]\left(\frac{e_t}{d_t}\right)^{\frac{q}{2}} I_q\left(2\sqrt{d_t e_t}\right) \tag{21}$$

$$\text{where} \quad c \equiv \frac{2\alpha}{\sigma^2(1 - \exp[-\alpha])} \tag{22}$$

$$
\begin{aligned}
d_t &\equiv cr_{t-1}\exp[-\alpha] && (23)\\
e_t &\equiv cr_t && (24)\\
q &\equiv \frac{2\alpha\theta}{\sigma^2} - 1 && (25)
\end{aligned}
$$

and where $I_q(.)$ is the modified Bessel function of the first order.

Both models yield affine term structures. In other words, the yield at time t on any $T-t$ maturity discount bond, $R(t,T)$, is a linear function of the instantaneous interest rate:

$$
R(t,T) = \frac{1}{T-t}\{B(t,T)r_t - \log A(t,T)\} \qquad (26)
$$

for a pair of functions, $A(t,T)$ and $B(t,T)$. In the simple case in which agents are risk neutral so the price of risk is zero, the functions $A(t,T)$ and $B(t,T)$ for the Vasicek and Cox-Ingersoll-Ross models are:

1. Vasicek Model

$$
\begin{aligned}
B(t,T) &= \frac{1-\exp[-\alpha(T-t)]}{\alpha} && (27)\\
A(t,T) &= \exp\left[(B(t,T)-(T-t))\left(\theta - \frac{\sigma^2}{2\alpha^2}\right) - \frac{\sigma^2}{4\alpha}B(t,T)^2\right] && (28)
\end{aligned}
$$

2. Cox-Ingersoll-Ross Model

$$
\begin{aligned}
B(t,T) &= \frac{2\exp[\gamma(T-t)]-1}{(\gamma+\alpha)(\exp[\gamma(T-t)]-1)+2\gamma} && (29)\\
A(t,T) &= \left(\frac{2\gamma\exp[(\alpha+\gamma)(T-t)/2]}{(\gamma+\alpha)(\exp[\gamma(T-t)]-1)+2\gamma}\right)^{\frac{2\alpha\theta}{\sigma^2}} && (30)\\
\text{where}\gamma &\equiv \sqrt{\alpha^2+2\sigma^2} && (31)
\end{aligned}
$$

To formulate a likelihood for a sample of finite maturity interest rates sampled at discrete intervals of time, one may infer the underlying interest rate, r_t, by inverting the relationship in equation (26) and then evaluate the densities in equations (18) and (21) multiplied by Jacobian adjustment terms to take account of the change of variable.

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